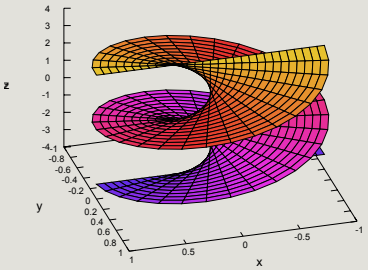

MATH 211: Multivariable Calculus

Classroom: SMUD 206

Instructor Info

- Dr. Miriam Kuzbary
- Pronouns: she/her/hers
- Call me: Dr. Kuzbary or Miriam
- Office Hours: TBA after survey
- Office: SMUD 013
- Course Website: On Moodle
- mkuzbary@amherst.edu

Course Info

- Prereq: A grade of C or better in MATH 121, placement into MATH 211, or consent of the Department.
- Mon, Wed, Fri
- 11:00am-11:50am
- SMUD 206

About

Though you have worked hard to learn calculus before, you likely only worked with functions of one variable. Very few systems we will ever encounter in life only involve a single variable; the vast majority of settings involve multiple variables, such as predicting a rainstorm, understanding inflation, or proving Maxwell's equations! In this class, we will develop the differential and integral calculus you have already encountered into multivariable tools.

What is the point of this class?

Calculus can be thought of as the study of continuous change. With derivatives, some of the things we can do is understand instantaneous change, approximate difficult functions, and more precisely optimize outcomes. With integrals, we can use many small, incomplete pieces of information to understand information about a whole system, such as averages, areas, and even expected values. These two mathematical ideas are connected by the fundamental theorem of calculus.

This class is a generalization of single-variable calculus to the multivariable setting and is a fundamental tool for using mathematics to understand the world around us and systems in it. More than that, Multivariable Calculus is full of deep, surprising, theoretical results which are interesting in their own right.

What you'll learn along the way

Throughout this semester, you should work to make progress towards the following goals:

- Demonstrate fluency with computational techniques in differential and integral calculus in several variables,
- Develop your geometric intuition for working in two and three dimensional space,
- Deepen your understanding of what concepts in calculus intuitively mean and how they apply to a variety of situations,
- Compare and contrast theorems in multivariable calculus relating derivatives and integrals to the fundamental theorem of calculus in the single-variable world, and
- Expand your independence as a math learner and practitioner.

A few things to help you be successful

Inside of Class

During our lecture time we will have many conversations about what we are learning, so come to class expecting that you will be both contributing to the discussion and taking away something interesting to think about. We are creating this class together. There will be many opportunities to work with your peers in groups and I will regularly solicit your input. Our class time is your opportunity to think out loud, make mistakes, and ask questions!

As a member of the Amherst College community, I am committed to creating a learning environment in which all of my students feel safe and included. Because we are individuals with varying needs, I am reliant on your feedback to achieve this goal. To that end, I invite you to enter into dialogue with me about the things I can stop, start, and continue doing to make my classroom an environment in which every person in it (including students, QFellows, graders, and myself) feels valued and can engage actively in our learning community.

Learning new mathematics takes a lot of work and often takes time to process and internalize. As with any math class, simply attending lectures will not be enough to succeed in the course. It is important you spend time outside of class actively reading the course textbook and that you study regularly, regardless of whether you have an assessment looming on the horizon. It is essential that you start your homework early enough to be able to ask questions about it in office hours and have conversations about it with your peers in class.

Since school during lockdown had a mix of delivery styles and many students report their attention spans and ability to engage in lecture has gone down in recent years, here is some advice on how to get the most out of our time in class together:

FAQs

How many hours should I expect to spend each week on this class?

Our class meets for 3 hours a week, so you should expect to spend at least 9 hours per week on this course. One way to break this up is for each hour of class time, spend one hour reviewing your notes, one hour doing the exercises, and one hour doing the problems.

What if I have a hard time with graphing functions or visualizing objects in space?

One of the best things about taking a class is having support to get better at things that are difficult! Spatial reasoning is something that can be improved with practice, and I promise by the end of the semester you will find these things easier than at the start.

How often should I come to office hours?

As often as you can, even if you don't have your own question! You might collaborate with classmates or learn from their questions.

What if I get sick?

If you are showing symptoms of a contagious illness and/or testing positive for COVID-19 on a regular class day, please follow the Amherst College *protocols* and do not come to class. If it is an exam day, please email me immediately to schedule a makeup exam.

- Eliminate distractions as best you can while you participate in class and work on this course outside of lecture.
  - Our brains are not usually physically able to "multitask," i.e. do multiple things in parallel. Instead, our brains switch very quickly between tasks. This means when you have multiple things going on when you are trying to learn a new thing, your brain cannot as easily store things you're learning in your long-term memory even though you feel like you are paying attention. For peer reviewed research about this in the study of cognition, see *Mayer and Moreno 2003*, *Junco 2012*, and *Junco and Cotten 2012*.
- Take notes during class, and after class summarize or outline the notes for yourself in a separate document.
- Work problems along with the lecture, even if you are going through your notes later. You get more out of doing math than you do watching math!
- Answer questions I ask in class, even if you're not confident your answer is correct! Talking through things is a good way to clarify ideas.
- Ask questions throughout the lecture!
- If you are unable to attend lecture at the normal time, get class notes from your classmates, write down questions you have, and email them to me.

Outside of Class

Regularly check both your Amherst email address and the course Moodle page to keep informed of any announcements, deadlines, assignments, syllabus adjustments, or policy changes made during scheduled classes.

This class and the math department have many resources for you to help you succeed. Your first resource is other students in the class; you should connect with other students early in the semester to form study groups, and ask questions to each other. Secondly, there are many office hours for you to attend. Office hours are times set aside for students to drop in, ask questions, and work through material with other students. There are TAs assigned to this course as well as QFellows with regular office hours. The QCenter is also open regularly for drop in help. Finally, I have office hours and am available by email.

Some other things to try:

- Come to office hours regularly, even if you don't have your own question! Someone else might ask a question that is relevant to you.
- Summarize or outline the definitions, propositions, theorems, and conjectures for yourself while you are reading the text.
- Try to solve problems from class for yourself without looking at the lecture notes, then, compare the solution you came up with to the solution we did together or you found in the book! Even if your solution is very different or you could not finish it your way, you will likely learn a lot from the experience.
- Discuss ideas from class with your peers in a respectful way, making sure everyone involved in the conversation is able to speak and work through problems together.
- Google strategies for succeeding in mathematics classes! There are many problem-solvers in the world, and we all have different perspectives on how to effectively learn mathematics and communicate it in a useful way.

Required Text

Multivariable Calculus, 8th edition by James Stewart. Copies of the textbook and its solution manual are on reserve in the Science Library.
Stewart's Single Variable Calculus and its student solution manual are also on reserve for your reference.

The Amherst College Honor Code

The Amherst College Honor Code applies to this course. It is your responsibility and mine to be familiar with and uphold *all aspects* of this code, including the Statement of Intellectual Responsibility, the Statement of Respect for Persons, the Statement of Freedom of Expression and Dissent, and the Statement of Student Rights.

How to earn a specific grade in this class

Your final grade in the class will be computed by:

Homework	17%
Highest grade midterm	20%
Next highest grade midterm	20%
Lowest grade midterm	10%
Final Exam	30%
Effort	3%

Passing grades will follow the standard scale:

A+	[97%-100%]	C+	[77%-80%)
A	[93%-97%)	C	[73%-77%)
A-	[90%-93%)	C-	[70%-73%)
B+	[87%-90%)	D+	[67%-70%)
B	[83%-87%)	D	[63%-67%)
B-	[80%-83%)	D-	[60%-63%)

A final course grade of less than 60% will result in a final letter grade of F. Curving is at my discretion, and if it happens will only happen at the end of the course once all assessments have taken place.

Homework

Mathematics is not a spectator sport! For example, if you are trying to learn how to swim, watching someone swim and explain swimming technique to you is certainly not going to be enough to prepare you to jump in the water and win a race. Math is a physical skill that takes time and practice to improve, and you might even not notice your progress as it is happening. Thankfully, we have an entire semester to work on this material together!

That said, doing homework and attempting your own ideas is the most important part of the course. Think of it this way: if what you are trying to do is solve problems and write your solutions in a way other people fluent in the course material can understand, then you should practice doing exactly that! It is important that you try as many problems as you can by yourself before consulting other sources. Resist the urge to search the internet or ask your friends who have already taken the class for solutions; this is not the way to learn the material well. *Some struggle is expected and is necessary for learning mathematics.*

Working in groups and talking through your ideas is a great thing to do, and a skill that will be invaluable throughout your mathematical journey. However, your homework write-ups must be written individually. Copying or paraphrasing the work of others is plagiarism and is *a violation of the honor code*. If you happened to work on an assignment with other people and you all came up with the same solution, please write the name(s) of the other students involved.

Finally, *do not give unsolicited answers to your classmates*. Again, part of the process of learning mathematics is the struggle itself, therefore

- do not rob your colleagues of the opportunity to figure things out, and
- remember that just because you think you have a solution does not mean that solution is correct!

The ideal discussion of the homework is one where everyone involved has tried the problem already, and everyone in the discussion is given space to try out their ideas.

There will be one homework assignment per week, due on *Tuesdays by 11:59pm* through Gradescope. I will not accept homework in person, you must scan it and turn it in online. Homework turned in after the due date will not be accepted (see the special arrangement section of the syllabus for extension policies). In order to make sure your work is organized well, please make sure you write legibly and label the problems in the same order as listed in the assignment.

Exams

There will be three midterm exams in this course. Each one will occur during our usual class time and will take 50 minutes. Our final exam will be cumulative and 3 hours long. The date and time of it will be announced closer to final exam week.

All exams in this course are individual, closed book, and closed note. No electronic devices are allowed. Mobile phones and smart watches must be put on silent and placed in a closed bag or on the floor.

Midterm Exam 1	Friday, September 29
Midterm Exam 2	Friday, October 27
Midterm Exam 3	Friday, December 1
Final Exam	TBA

Special arrangements

Extensions and Make-Ups

If for some reason you cannot hand in your homework in time, you can request up to TWO extensions during the term with no questions asked. You must contact me no later than the day before the due date to let me know. If you have a religious holiday on the same date as an exam in this course, let me know within the *first two weeks of the semester*.

Accommodations

I strive to support all students so please come meet with me if you have any questions or concerns about your engagement and success in this course. Students seeking general disability services and/or accommodations should contact Accessibility Services. You can reach them via email at accessibility@amherst.edu, or via phone at 413-542-2337. Once you have your accommodations in place, I will be glad to meet with you privately during my office hours or at another agreed upon time to discuss the best implementation of your accommodations. For more information, please visit the Accessibility Services website.

What you'll be doing

The following times and topics are tentative and may shift slightly to foster a more effective learning environment. Nothing will be made due earlier than indicated but some things may be pushed back or eliminated altogether, depending on time. All changes will be announced in class and posted on the course website.

MODULE 1: 3-Dimensional Geometry

Week	Topics we'll explore	Suggestions for how to prepare
Week 1	• The 3-dimensional Cartesian coordinate system and vectors	• Read sections 12.1 and 12.2 in the textbook, working along with problems as you go without looking at solutions or getting help until you have made an effort on your own • Draw some sketches of vectors in 3D by hand
Week 2	• Ways to combine vectors • Constructing subsets of 3D space	• Read sections 12.3, 12.4, and 12.5 in the textbook, working along with problems as you go without looking at solutions or getting help until you have made an effort on your own

Week 3

- Lines and planes
- Understanding 3-dimensional pictures with 2-dimensional slices

- Read sections 12.5, 12.6, 13.1, and 13.2 in the textbook, working along with problems as you go without looking at solutions or getting help until you have made an effort on your own
- Come up with your own equations describing subsets of 3D space and sketch the corresponding sets by hand
- Put equations describing subsets of 3D space into Wolfram Alpha and rotate the view to better understand those subsets

Week 4

Exam 1
Friday, 9/29

- Lines and planes
- Understanding 3-dimensional pictures with 2-dimensional slices

- Read sections 13.2, 13.3, 14.1 in the textbook, working along with problems as you go without looking at solutions or getting help until you have made an effort on your own
- Study for Exam 1

MODULE 2: Functions of several variables, taking their derivatives, and what those derivatives mean

Week 5

- Multivariable functions
- The precise definition of a limit
- Why a limit should not depend on the path you take
- What it means to be continuous
- Partial Derivatives

- Review from single-variable calculus: definitions of limits and continuity and derivative formulas
- Read sections 14.1, 14.2 and 14.3 in the textbook, working along with problems as you go without looking at solutions or getting help until you have made an effort on your own

Week 6

- Different types of derivatives and what they mean
- Tangent planes and approximating multivariable functions
- The chain rule in 3D

- Review from single-variable calculus: Tangent lines, linearization, and the chain rule
- Read sections 14.4, 14.5, and 14.6 in the textbook, working along with problems as you go without looking at solutions or getting help until you have made an effort on your own
- Sketch some graphs of functions in 3D from different viewpoints and draw a couple of tangent planes at a few different points on the graph. How well do the different tangent planes approximate the function?
- Sketch some graphs of functions in 3D from different viewpoints and draw the gradient vector at a few different points on the graph. How does the length and direction of this vector compare to the graph of the function?
- Think about the difference between the graph of a function and a subset of 3D space described by an equation (or some number of equations). Can you come up with concrete examples?

Week 7

- Finding maxima and minima with partial derivatives
- Using gradients to look for maxima and minima with the method of Lagrange multipliers

- Review from single-variable calculus: Optimizing functions
- Read sections 14.7 and 14.8 in the textbook, working along with problems as you go without looking at solutions or getting help until you have made an effort on your own
- Try optimizing (i.e. looking for maxima and minima) the same function using both methods. In which situations are partial derivatives more appropriate and in which situations are Lagrange multipliers easier or more efficient?

MODULE 3: Taking integrals of different types of functions involving multiple variables

Week 8
Exam 2 Friday, 10/27

- Taking double integrals in Cartesian coordinates
- Taking double integrals in polar coordinates

- Review from single-variable calculus: Integral formulas, u-substitution, integration by parts
- Read sections 15.1, 15.2, and 15.3 in the textbook, working along with problems as you go without looking at solutions or getting help until you have made an effort on your own
- Sketch *each region* you are taking an integral over, the bounds for these integrals are usually more subtle than they first appear!
- Study for Exam 2

Week 9

- What double integrals are good for
- Taking triple integrals in Cartesian coordinates
- Taking triple integrals in cylindrical coordinates

- Review from single-variable calculus: U-substitution and trigonometric substitution
- Read sections 15.4, 15.6, and 15.7 in the textbook, working along with problems as you go without looking at solutions or getting help until you have made an effort on your own
- Sketch *each region* you are taking an integral over, the bounds for these integrals are usually more subtle than they first appear! This is particularly important when you are changing coordinate systems or changing the order of integration

Week 10

- Taking triple integrals in Spherical coordinates
- Changing coordinate systems/variables in several dimensions
- Line integrals of scalar-valued functions
- Line integrals of vector fields

- Review from single-variable calculus: Parametric curves
- Read sections 15.8, 15.9, 16.1, and 16.2 in the textbook, working along with problems as you go without looking at solutions or getting help until you have made an effort on your own
- Sketch *each region* you are taking an integral over, the bounds for these integrals are usually more subtle than they first appear! This is particularly important when you are changing coordinate systems or changing the order of integration
- Come up with some of your own vector fields and draw them on a subset of 2d or 3D Cartesian coordinates.

MODULE 4: Relating derivatives and integrals involving several variables

Week 11

- The fundamental theorem of line integrals
- Green's theorem

- Review from single-variable calculus: The fundamental theorem of calculus
- Read sections 16.1, 16.2, 16.3, and 16.4 in the textbook, working along with problems as you go without looking at solutions or getting help until you have made an effort on your own
- Sketch each region you are applying Green's theorem to

Week 12 THANKSGIVING BREAK

Week 13
Exam 3
Friday, 12/1

- Operations on vector-valued functions: gradient, curl, and divergence
- Oriented surfaces
- Surface integrals

- Read sections 16.5, 16.6, and 16.7 in the textbook, working along with problems as you go without looking at solutions or getting help until you have made an effort on your own
- Practice writing down parametric equations for different surfaces, and make sure to sketch the surfaces! Feel free to use computer tools to make visualization easier, but make sure you practice at least a little bit by hand.
- Study for Exam 3

Week 14

- Surface integrals
- Stokes' Theorem
- The divergence theorem

- Read sections 16.7, 16.8, and 16.9 in the textbook, working along with problems as you go without looking at solutions or getting help until you have made an effort on your own
- Sketch out examples illustrating what the theorems this week actually mean

Week 15

- Summarizing the course
- Reviewing course material
- Making up missed class day(s)

- Read section 12.10 in the textbook, working along with problems as you go without looking at solutions or getting help until you have made an effort on your own
- Study for Final Exam (comprehensive)

FINAL EXAM WEEK

Important Dates

September 5	First Day of Classes
September 14	Last Day of Add/Drop
October 9-10	Midsemester Break
November 20-24	Thanksgiving Break
December 13	Last Day of Classes
December 16,18-21	Final Exams

This syllabus provides a general plan for the course; deviations may be necessary. You are responsible for all of the information in this syllabus, so please read it carefully and refer back to it regularly.