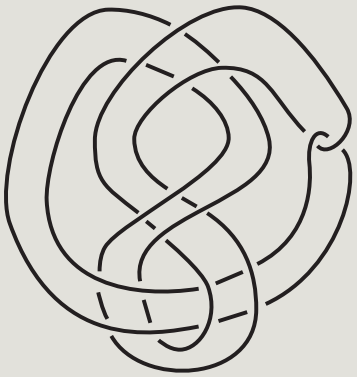

MATH 279: Introduction to Knot Theory

Classroom: SMUD 014

Instructor Info

Dr. Miriam Kuzbary

Pronouns: She/Her/Hers

Call me: Prof. Kuzbary or Dr. Kuzbary

Office Hours:
Drop-In: MF 9:30-10:30am, W 4-5pm
Appt: M 3:30-4pm, Thu 3-4pm

Office: SMUD 013

Course Website: On Moodle

mkuzbary@amherst.edu

Course Info

Prereq: A grade of C or better in MATH 271 or 272, or instructor consent.

Mon and Wed

2:00pm-3:20pm

About

Welcome to Introduction to Knot Theory! Knot theory is a rich and active area of research involving questions of interest both to mathematicians and to researchers outside of mathematics, and many of these questions boil down to a single essential query: how can we tell when two knots are different?

What you'll learn along the way

Throughout this semester, you should plan to work towards the following goals:

- Prove theorems, propositions, and corollaries about knot theory topics,
- Perform knot theory computations,
- Develop your geometric intuition and fluency in switching between algebraic and geometric understanding of the same problems,
- Become comfortable in introductory topics in low-dimensional topology (dimensions 0, 1, 2, 3, and 4).

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Mutual Expectations

Learning new mathematics is a difficult thing to do, and takes a lot of work and time! That said, you are *completely capable of mastering this material if you are willing to put in the time, attention, and strategic effort into it.*

However, mathematics is not a spectator sport! For example, if you are trying to learn how to swim, watching someone swim and explain swimming technique to you is certainly not going to be enough to prepare you to jump in the water and win a race by itself. Math is a physical skill that takes focused practice over time to improve. You might even not notice your progress as it is happening. This is a hard course, but I'm here to help you succeed!

Some of the things you can expect from me:

- The most important thing to me is that you learn something in this class, and that will require work from both of us!
- I am committed to creating a learning environment in which all of my students feel safe and included. Our classroom is a living community that will change and grow based on our participation and interaction in it, and as its facilitator I will constantly work to help it be a place of trust, curiosity, and mutual respect where learners with many different communication styles, needs, and goals can participate meaningfully. I hope you will, too!
- I have been teaching and tutoring for quite some time (15 years). Students from a huge range of backgrounds and with many different kinds of preparation for college math have succeeded both in my classes and others I have helped them with. This means when I make choices in the classroom or suggest something for you to try, it's because I have seen it work for multiple people.
- I will always assume you are acting in good faith and with good intentions, and I hope you will assume that of me and your fellow classmates as well!
- I will respond to your emails within two business days (meaning Monday through Friday, excluding holidays), during normal business hours (9am-5pm) unless it is an emergency, in which case I will do my best to respond sooner. You can always email me questions at any time, including about specific problems or proofs!
- I will do my best to clearly communicate expectations and standards in this class, and will regularly solicit feedback from you on how it's going,

- I will also work to allow for as many different students to speak in each class session as possible, which will involve a few strategies. Some of these include: allowing for 1-2 minutes after each question so that all students have time to process the question and think about how they'd like to answer, asking you to discuss a question in pairs for 3-4 minutes and then calling on a pair to tell me what they talked about, or waiting to call on someone until at least 4-5 hands are raised to talk.

Some of the things I will expect from you:

- Approach this course with curiosity and commitment: some struggle is expected and is necessary for learning mathematics! I will *never ask you to do something I don't believe you are capable of doing*, and sometimes we learn and grow the most by trying things that feel difficult.
- Trust yourself that you are capable of meeting the expectations I have for you in this course. Mistakes are good: they help you clarify how you are understanding the material and where you have more room to grow!
- Use our class time effectively and treat your fellow class members with respect and care. This can include asking questions, even if they feel silly, because if you have a question there are usually multiple other students with the same question who aren't speaking up! Participating in class in a respectful way also includes listening to your classmates, stepping up to take space when appropriate, and allowing other class members to take up space when appropriate.
- Assume good intentions from other students and myself, but please acknowledge in a timely fashion if you are feeling hurt by information in the class and/or the way it is shared.
- Try not to make assumptions and ask questions to learn more about other perspectives and strategies from your peers and me, especially those that are different from your own.
- If something is going on with you that affects your participation in this class, please tell me as early as possible!
- Read the entire syllabus carefully, familiarize yourself with the entire course Moodle page, and check your email every day, even if you take a business day or two to respond to non-urgent emails.
- If what you've done so far in the course isn't giving you the results you want, be open to changing how you are studying, doing homework, participating in class, or participating in office hours(among other things) and please come to office hours or schedule a meeting with me to chat about it!
- If you have a concern about a grade you've earned on an assignment, reach out to me AS SOON AS POSSIBLE. I won't be able to address regrade requests or concerns about a grade if I get them more than one week after you have received the grade.
- Keep track of deadlines and assignments, turn things in on time, and take in-class assessments on time.
- Follow assignment instructions carefully.

Illness Policy

If you are showing symptoms of a contagious illness and/or testing positive for COVID-19 on a regular class day, please follow the Amherst College *protocols* and do not come to class. If it is an exam day, please email me immediately to schedule a makeup exam. I (and some of your classmates) are high risk, so if you are sick please rest and protect yourself and our community!

Abstract Math and Proofs

Let's get the scariest part out of the way: there will be proofs in this class! That said, since this is a 200-level elective, I am very aware that many of you are pretty new to writing proofs in college and am here to help you improve your proofs.

This is a pretty abstract course, but remember abstraction is just a way to cover many similar situations with the same framework! And we will see lots of examples to illustrate abstract concepts.

Why proofs? What is the point of proof and why do we emphasize them in mathematics?

When you've proven something, you not only know it's true, but additionally you understand how or why it's true. So if you're interested in deep and conceptual understanding of the tools you're using (as opposed to just being handed a formula, being told to trust that it works and to not question where it came from), proofs become very useful.

We would also like to emphasize that even if you're only interested in applying the tools that we learn, it will not be enough to just learn a few "recipes" and to become proficient with them. This is because when dealing with hard problems in the real world, it will almost never be the case that some formula or recipe you learned in any of your classes will apply directly to your situation. You'll have to tweak, alter, or even completely replace those recipes because the real world is complicated! And to do this, you really need to know why the recipe works and what the logic is behind it. If you treat a formula like magic, you won't be able to dissect it later on and alter it when you need something a bit different!

Our course will sometimes feel like we're doing two very different things at once: (1) practicing with the recipes of knot theory and doing computations, and (2) justifying those recipes through proof and studying abstract mathematical concepts. Try

to think of these two halves as being deeply related to one another; in particular, learning the theory behind the recipe is essential to honing our practice and our ability to apply knot theory in the real world.

The components of this course and their purpose

In-class work, discussion, and lecture

During our lecture time we will have many conversations about what we are learning, so come to class expecting that you will be both contributing to the discussion and taking away something interesting to think about. We are creating this class together. The in-class portion of this course is time to engage with material together, ask questions, and do practice problems and proofs with immediate feedback from me. Our class time is also your opportunity to think out loud, make mistakes, and ask questions!

I encourage you to stop me during the lectures and ask questions. If you are feeling lost, it is extremely likely there is someone else sharing the same feeling. Asking your question ensures the whole class gets to benefit from the answer!

I like to stop from time to time and ask around questions about the material, please do not feel intimidated! It is a way to encourage thinking through things in a different way, and for me to check in with how you all are connecting with the material in real time.

Class participation is part of the Effort portion of your grade. If you are quiet by nature, don't worry; as long as you attend class the majority of the time, pay close attention, and do the homework, you will get full Effort credit. That said, even though this is a lecture course, class should be interactive, and participating in classroom discussion helps you learn the material. In addition, when I ask a question to the class, I'm usually expecting an answer. If you have even a vague idea of how to start approaching the question, please share it!

Studying and learning outside of class

We only spend 3 hours in class each week together, and according to the Federal Credit Hour standard that means you should expect to spend at least 2 hours outside of class per week for each credit hour of class. Therefore the time you spend outside of class on this material is the single biggest component of the course! To help support you with this work outside of class, I will occasionally post videos of extra problems and proofs and will post selected homework solutions.

The time outside of class you spend learning and studying is how you develop your own perspective on the course material, deepen your understanding, and strengthen your skills. It is also important that you check in with yourself regularly on how the class is going, and part of your Effort grade will involve occasional reflection assignments.

Reading Assignments

College courses have significantly fewer contact hours than high school courses; this is because you are expected to both attend class and read material outside of class in order to get the understanding you need to master the material.

It would be amazing if we could all learn something once and have it stick forever, unfortunately human brains do not work this way! We learn best and most deeply from encountering and engaging in material multiple times, in multiple different ways. This is part of why it is so crucially important that you both come to class and *do the assigned reading each week.* These components of the course support each other in helping you reach fluency with the topics in this course!

Homework Problems

Homework gives you an opportunity to get feedback on your demonstrated understanding of course material outside of an exam. That said, you would probably prefer to make mistakes on the homework and learn how to fix them before a quiz or exam than make a mistake for the first time on a quiz or exam! This means homework will be most beneficial to you if you study the material and look at examples *before you start the related homework*, and push yourself to do the homework in a closer to exam-style environment! Try not to flip back to example problems unless you have given a problem a serious try without any help, but please don't be afraid to ask for help once you've given it a try and don't feel comfortable with it yet!

Your homework consists both of reading the relevant sections of the book and of doing the assigned problems. (Only the written work counts directly in your grade, but I expect you to do both.)

Homework grading will be more general feedback than quizzes and exams; this is intentional to help you feel more comfortable trying out the material for the first time. On quizzes and exams, you have had more time and practice with the material. After each homework assignment, I will post solutions to selected problems. *Once the solutions have been posted, I will not accept that homework assignment late, even if you haven't used up your two allowed extensions.*

Working in groups and talking through your ideas is a great thing to do, and a skill that will be invaluable throughout your mathematical journey. However, the homework you turn in must be written *by you*. Copying or paraphrasing the work of others is plagiarism and is *a violation of the honor code*. This includes using ChatGPT, Chegg, CourseHero, Wolfram Alpha,

and similar websites and programs. If you happened to work on an assignment with other people, please write the name(s) of the other students you worked with at the top of the assignment.

It has become very common for students to use GenAI (artificial intelligence or machine learning tools such as ChatGPT or Dall-E 2) on homework assignments in ways other than simply asking for answers to homework problems. This practice may not be particularly helpful for preparing you for the quizzes or the exams; however, if you still decide to use it for homework, you must properly document and credit how you used it. This includes using it to brainstorm or generate code, outlines, or other text for you. Note that GenAI tools often "hallucinate" incorrect statements or computations, so be careful to double check its output if you rely on it to study! This is especially crucial for knot theory as it is a younger field than algebra, geometry, or calculus, and the data about it that GenAI is training on is much smaller.

Sample Citation: Chat-GPT-3. (YYYY, Month DD of query). "Text of your query." Generated using OpenAI. https://chat.openai.com/
Note: Material generated using other tools should follow a similar citation convention.

Finally, if you talk to other students about the homework problems *do not give unsolicited answers to your classmates*. The ideal discussion of the homework is one where everyone involved has tried the problem already, and everyone in the discussion is given space to try out their ideas.

A bit of advice for the homework in this class:

Individual homework problems, whether computational or theoretical, will require more strategizing and scratchwork than you may be used to in non proof-based courses. As a rule of thumb, try to write your work so that someone in the class who is sort of following along, but is confused about a lot of things, can understand your answer without having to look at the textbook or lecture notes. Just like in your humanities classes, think about how to make your thoughts clear to a reader on first reading.

• *Computations*: Please show every step and justify all your work, *using words* along the way. There are graded examples on Moodle.

• *Proofs*: I'll model lots of proofs in class. So follow my models if you want to know what I and your professors in other math classes will expect you to do! There are graded examples on Moodle and a rubric.

• *All problems*: Please use complete sentences and well-written paragraphs.

Of course, equations will usually appear, too, and you can certainly use abbreviations and standard mathematical shorthand. But fundamentally, solving any mathematics problem, whether computational or theoretical, is about making an argument using *words*.

- Start each problem thinking first about what it is you are trying to do (write it down!) and then thinking about what information you have to start with.
- Start the homework *the day it is assigned*, even if we haven't covered all of the material for the assignment. You can still work on the problems related to material we have covered!
- Study for your homework: look over your notes from class and the reading before looking at homework problems. It is more important to focus on learning the material well using the homework as practice than to just focus on finishing every problem!
- Space out studying and working on your homework throughout the week; cramming can increase stress and decrease our long term retention of new material. Try out *the study cycle (link)*!
- Since Knot Theory is very geometric, it is extremely rare to come across a proof in this class that shouldn't start with you drawing something! It's ok if your drawing doesn't look pretty; the point is to have a visual representation of the problem to start with rather than trying to only visualize the problem in your head.
- Start on scratch paper first to organize your thoughts and try things out. Then, once you think you have a solution, write it up carefully to turn in!
- Justify all your logical steps in a problem, and use the grading example on Moodle as a guide for the level of explanation that I am looking for.
- You will get more out of office hours if you start thinking about the material and where you might be confused before you come. Office hours hours are finite, so preparing for them can help them be more useful for you!

Exams

There will be 2 midterm exams in class and no final exam.

Midterm Exam 1	Wednesday, March 5
Midterm Exam 2	Wednesday, April 16

You will be allowed one 8.5' x 11" page of notes (you can use the front and back of the paper) which you must turn in with your exam. I strongly suggest making your own page of notes rather than sharing with another student, as making your own note sheet will help you organize your thoughts in the best way for you and usually helps you study. Aside from that one page, the exams for this class will be closed book and closed note. Calculators, cell phones, laptops, ipads, etc. are not permitted in exams.

The kind of deep studying required for an exam allows you to engage in the material in a different way and learn the material more thoroughly. Think about it this way: would you study harder for a class with no exams or a class with exams?

Finally, exams provide valuable feedback for both of us on specific things you are understanding and not understanding in this course, and can help both of us make an informed decision about how the class is going.

Final Project and Presentation

The research project is your opportunity to explore a topic related to the class that is relevant to your interests and the classes you have already taken so far. The project can be interdisciplinary if you would like it to be!

The goals are to learn something new, collaborate with classmates, communicate what you've learned, and have fun. Near the end of the term, each person will give a short presentation to the class and each person will write a report. While each project is individual, you will have a small group to support your work!

There will be a short presentation the last week of class on your topic and intermediate assignments incorporating peer feedback and workshopping. The final, written project will be due the last day of finals week.

Keep an eye out later on in the semester for more detailed information on the project.

Office Hours

Office hours are your time to have smaller group interaction and feedback with us to clarify concepts, ask questions, and get help if you are stuck on a problem. I have a mix of drop-in and by appointment office hours so that you can choose what kind of environment you'd like! Please come to office hours as often as you'd like, and please know that *all of us* need help sometimes! I will never judge you or think less of you for coming to office hours or for the things you would like to talk about in office hours or questions you ask. I truly hope to see all of you in office hours!

You can think about going to office hours like meeting with a coach or a trainer: I can demonstrate lifting a weight for you, I can help you make a plan to improve your lifting, but ultimately, if you don't also actually lift the weight yourself you won't get the benefits from lifting weights! Office hours are for helping you navigate problem solving and learning the material in the course, which is not the same thing as simply telling you the solutions to homework problems. Office hours supplement the rest of the components of this course, but they *cannot replace any other component of the course*.

Required and Recommended Texts

All required textbooks at Amherst are now provided by the college! You will receive an email from customerservice@efollett.com notifying you that course materials are ready for pickup.

Please bring your Student ID to the textbook distribution center at the Alumni House to collect your course materials. If you drop the course on or before the last day of the add/drop period, you must return the print materials to the same Alumni House location.

Distribution Dates: Starting January 20 *Location: Alumni House*
Hours Monday - Friday, from 10 a.m. to 4 p.m., January 25-26, February 1-2 9 a.m.-3 p.m.

Required texts

The Knot Book: An Elementary Introduction to the Mathematical Theory of Knots by Colin C. Adams,
Knot Theory by Charles Livingston (only available new as e-book).

Other texts which might be useful

Teach Yourself How to Learn: Strategies you can use to ace any course at any level by Saundra Yancy McGuire.
This is an excellent book to use as a reference for study strategies in addition to talking to me and the Strategic Learning Center, and especially if you find yourself spending more than 10 hours a week on this class outside of class or stressing out regularly about this class!

Add/Drop and Attendance Policy

It is very difficult to catch up in a math class if you miss classes, even the first couple of days of the semester. We will start out the semester with some tricky, abstract ideas and trying to catch up if you miss the topics the first time in class is unnecessarily difficult. If you're interested in adding this class, you should attend every class the first week.

Our time together in class is very important and there is quite a lot of data that attending class regularly strongly correlates with success in the course (see Credé, Roch, and Kieszczynka 2010).

As a result, I will take attendance in this class. If you have at least three unexcused absences (which is over 10% of our class meetings for the semester since we only meet twice a week) you must schedule an individual meeting with me to discuss your absences. This may or may not be paired with a grade penalty.

The Amherst College Honor Code and Course GenAI Policy

The Amherst College Honor Code applies to this course. It is your responsibility and mine to be familiar with and uphold *all aspects* of this code, including the Statement of Intellectual Responsibility, the Statement of Respect for Persons, the Statement of Freedom of Expression and Dissent, and the Statement of Student Rights.

Generative AI will not be allowed on any in-class assignment and if used will be considered a violation of the honor code. Honor code violations carry a variety of consequences depending on severity and frequency, and may include earning a 0% on a problem or entire assignment or exam.

How to earn a specific grade in this class

Your final grade in the class will be computed by:

Effort	3%
Highest Midterm Exam	30%
Lowest Midterm Exam	25%
Final Project	25%
Homework	17%

"Effort" is a combination of class attendance, class participation, and handing in problem sets. Final grades will be determined based on a holistic evaluation of your performance throughout the course. This may include, but is not limited to, mastery of both the proof-writing and computational components of the course, participation, improvement over time, and engagement with course materials. At the end of the semester, I as the instructor reserve the right to make adjustments to the final grade to reflect overall effort and learning in the course. Final course grades will be curved.

Any student who skips an exam for an unexcused reason or fails to hand in at least 30 homework problems *on time* over the course of the semester *automatically* gets an F in the class.

Special arrangements

Illness Policy

If you are showing symptoms of a contagious illness and/or testing positive for COVID-19 on a regular class day, please follow the Amherst College *protocols* and do not come to class. If it is an exam day, please email me immediately to schedule a makeup exam.

Extensions and Make-Ups

If for some reason you cannot hand in your homework in time or take a quiz on time, you can request up to TWO extensions during the term with no questions asked. You must contact me no later than the day before the due date to let me know. If you have a religious holiday on the same date as an assignment or exam in this course, let me know within the *first two weeks of the semester*.

Accommodations

I strive to support all students so please come meet with me if you have any questions or concerns about your engagement and success in this course. Students seeking general disability services and/or accommodations should contact Accessibility Services. You can reach them via email at accessibility@amherst.edu, or via phone at 413-542-2337. Once you have your

accommodations in place, I will be glad to meet with you privately during my office hours or at another agreed upon time to discuss the best implementation of your accommodations. For more information, please visit the Accessibility Services website.

Electronics Policy

Please silence your phone and put it away during class. As many students find laptops distracting, if you plan to use a laptop during class please sit in the back row.

What we'll be learning about

Here is a large list of topics we will try to talk about this semester! It is very likely this list will metamorphose into something quite different by the end of the semester depending on the interests of the class. This list of topics won't grow, but may shrink!

Module 1: Knot Theory Basics

- Definition of a mathematical knot
- Equivalence of knots
- Ambient isotopy and planar isotopy
- Knot diagrams and projections
- Orientations
- The Reidemeister Moves
- Knot invariants
- Tricolorability
- Links
- Linking number

Module 2: How do we figure out which knot we have?

- Knot tabulation, knotinfo, and linkinfo
- Connected sum of knots
- Prime knots
- Crossing number
- Unknotting number
- Modular arithmetic (clock math)
- Mod p coloring
- Unknotting number
- Determinant of a knot
- Mod p rank of a knot
- Alexander polynomial

Module 3: Some geometric topology basics

- Surfaces and homeomorphisms
- Euler characteristic
- Genus
- The classification of surfaces
- Seifert surfaces
- Checkerboard surfaces
- Cut-and-paste topology (surgery)
- Knot complements
- 3-Manifolds

Module 4: Linear algebra and its surprising 4-dimensional consequences

- The Seifert matrix
- Seifert matrices and the Alexander polynomial
- Laurent polynomials
- Determinant of a knot
- Signature of a knot
- Slice knots
- Concordance
- Slice genus

- Ribbon knots

Module 5: Types of knots

- Torus knots
- Pretzel knots
- Twist knots
- Satellite knots
- Knot symmetries
- Thurston classification

Module 6: More polynomial knot invariants

- The Conway polynomial
- The Kauffman bracket
- The Jones Polynomial
- The HOMFLY-PT Polynomial

Module 7: Applications of knot theory

- DNA topology
- Synthesis of knotted molecules
- Chirality
- Statistical mechanics and knots

Important Dates

January 27	First Day of Classes
February 5	Last Day of Add/Drop
March 17-21	Spring Break
April 21-22	April Break
May 6	Last Day of Classes
May 9-15	Final Exams

This syllabus provides a general plan for the course; deviations may be necessary. You are responsible for all of the information in this syllabus, so please read it carefully and refer back to it regularly.