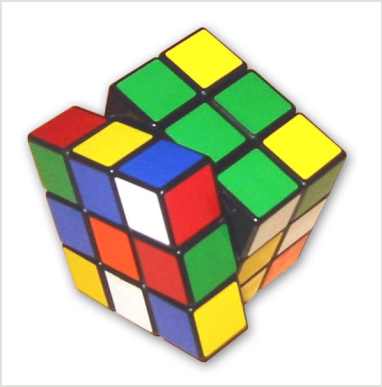

MATH 350: Groups, Rings, and Fields

Classroom:
Science Center E210

Instructor Info

Dr. Miriam Kuzbary

Pronouns: She/Her/Hers

Call me: Prof. Kuzbary, Dr. Kuzbary, or Dr. K.

Office Hours: M/W/Th

Office: SMUD 013

Course Website: On Moodle

mkuzbary@amherst.edu

Course Info

Prereq: MATH 211 and either MATH 271 or 272, or consent of the instructor.

Mon, Wed, Fri

10:00am-10:50am

SCCE E210

About

Welcome to Groups, Rings, and Fields! In this class we will begin the study of abstract algebra. Together we will learn deep ideas about algebra, formally and precisely prove results based on these ideas, and build a framework for ourselves connecting these ideas together.

What you'll learn along the way

The main area of mathematics covered in our course is called *Abstract Algebra*, which focuses on mathematical objects (groups, rings and fields) equipped with algebraic structures (operations) satisfying a set of axioms. Our course will focus on groups, which appear naturally in other fields, such as physics, chemistry and even art! For instance, groups help to model the idea of *symmetry* in physics, whereas rings and fields generalize the properties that we observe in certain classes of numbers, such us the rational numbers. You already have seen some of these objects (without giving it that name) in linear algebra. In this course we will study the axioms that rule groups, rings and fields, as well as their key properties using mathematical rigor (proofs). The main objectives of this course are for you to...

- Demonstrate fluency with the precise formal definitions of fundamental objects in the study of groups, rings, and fields as well as related definitions and theorem statements,
- Recognize patterns, formulate conjectures about those patterns, and prove or disprove said conjectures,
- Write clear and cogent proofs of the statements, propositions, and theorems contained in this course,
- Apply various notions and constructions of abstract algebra to different scenarios, and
- Build and solidify mathematical intuition, reasoning and the ability to connect different areas of mathematics.

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Mutual Expectations

Learning new mathematics is a difficult thing to do, and takes a lot of work and time! That said, you are *completely capable of mastering this material if you are willing to put in the time, attention, and strategic effort into learning how to do linear algebra.*

That said, mathematics is not a spectator sport! For example, if you are trying to learn how to swim, watching someone swim and explain swimming technique to you is certainly not going to be enough to prepare you to jump in the water and win a race by itself. Math is a physical skill that takes focused practice over time to improve. You might even not notice your progress as it is happening. This is a hard course, but I'm here to help you succeed!

Some of the things you can expect from me:

- The most important thing to me is that you learn something in this class, and that will require work from both of us!
- I am committed to creating a learning environment in which all of my students feel safe and included. Our classroom is a living community that will change and grow based on our participation and interaction in it, and as its facilitator I will constantly work to help it be a place of trust, curiosity, and mutual respect where learners with many different communication styles, needs, and goals can participate meaningfully. I hope you will, too!
- I will always assume you are acting in good faith and with good intentions, and I hope you will assume that of me and your fellow classmates as well!

- I will respond to your emails within two business days (meaning Monday through Friday, excluding holidays), during normal business hours (9am-5pm) unless it is an emergency, in which case I will do my best to respond sooner. You can always email me questions at any time, including about specific problems or proofs!
- I will do my best to clearly communicate expectations and standards in this class, and will regularly solicit feedback from you on how it's going,
- I will also work to allow for as many different students to speak in each class session as possible, which will involve a few strategies. Some of these include: allowing for 1-2 minutes after each question so that all students have time to process the question and think about how they'd like to answer, asking you to discuss a question in pairs for 3-4 minutes and then calling on a pair to tell me what they talked about, or waiting to call on someone until at least 4-5 hands are raised to talk.

Some of the things I will expect from you:

- Approach this course with curiosity and commitment: some struggle is expected and is necessary for learning mathematics! I will *never ask you to do something I don't believe you are capable of doing*, and sometimes we learn and grow the most by trying things that feel difficult.
- Trust yourself that you are capable of meeting the expectations I have for you in this course. Mistakes are good: they help you clarify how you are understanding the material and where you have more room to grow!
- Use our class time effectively and treat your fellow class members with respect and care. This can include asking questions, even if they feel silly, because if you have a question there are usually multiple other students with the same question who aren't speaking up! Participating in class in a respectful way also includes listening to your classmates, stepping up to take space when appropriate, and allowing other class members to take up space when appropriate.
- Assume good intentions from other students and myself, but please acknowledge in a timely fashion if you are feeling hurt by information in the class and/or the way it is shared.
- Try not to make assumptions and ask questions to learn more about other perspectives and strategies from your peers, math fellows, and me, especially those that are different from your own.
- If something is going on with you that affects your participation in this class, please tell me as early as possible!
- Read the entire syllabus carefully, familiarize yourself with the entire course Moodle page, and check your email every day, even if you take a business day or two to respond to non-urgent emails.
- If what you've done so far in the course isn't giving you the results you want, be open to changing how you are studying, doing homework, participating in class, or participating in office hours and math fellow hours (among other things).
- If you have a concern about a grade you've earned on an assignment, reach out to me AS SOON AS POSSIBLE. I and the graders won't be able to address regrade requests or concerns about a grade if we get them more than one week after you have received the grade.
- Keep track of deadlines and assignments, turn things in on time, and take in-class assessments on time.

Abstract Math and Proofs

For most of you, Groups, Rings, and Fields will be one of the more abstract math courses you will take at Amherst College. But although it might sound scary at first, abstraction is just a way to cover many similar situations with the same framework! And we will see lots of examples to illustrate abstract concepts.

A bit of advice for the homework in this class:

Individual homework problems, whether computational or theoretical, will require more strategizing and scratchwork than in non proof-based courses. As a rule of thumb, try to write your work so that someone in the class who is sort of following along, but is confused about a lot of things, can understand your answer without having to look at the textbook or lecture notes. Just like in your humanities classes, think about how to make your thoughts clear to a reader on first reading.

• *Computations*: Please show every step and justify all your work, *using words* along the way. There are graded examples on Moodle.

• *Proofs*: I'll model lots of proofs in class.So follow my models if you want to know what I and your professors in other math classes will expect you to do! There are graded examples on Moodle and a rubric.

• *All problems*: Please use complete sentences and well-written paragraphs.

Of course, equations will usually appear, too, and you can certainly use abbreviations and standard mathematical shorthand. But fundamentally, solving any mathematics problem, whether computational or theoretical, is about making an argument using *words*.

Start on Scratch Paper: Please take pride in handing in neat and organized written work in math courses. In particular, unless a given problem is a *totally* straightforward computation, *don't start working on a problem on the same piece of paper that you will hand in.*

Instead, start the problem on scratch paper, to figure things out before you write anything about that problem on the piece of paper that you will hand in.

The components of this course and their purpose

In-class work, discussion, and lecture

During our lecture time we will have many conversations about what we are learning, so come to class expecting that you will be both contributing to the discussion and taking away something interesting to think about. We are creating this class together. The in-class portion of this course is time to engage with material together, ask questions, and do practice problems and proofs with immediate feedback from me. Our class time is also your opportunity to think out loud, make mistakes, and ask questions!

I encourage you to stop me during the lectures and ask questions. If you are feeling lost, I am almost certain that there is someone else sharing the same feeling. I like to stop from time to time and ask around questions about the material, please do not feel intimidated, it is a way to encourage discussion!

Class participation is part of the Effort portion of your grade. If you are quiet by nature, don't worry; as long as you attend class devotedly, pay close attention, and do the homework, you will get full Effort credit. That said, even though this is a lecture course, class should be interactive, and participating in classroom discussion helps you learn the material. In addition, when I ask a question to the class, I'm usually expecting an answer. If you have even a vague idea of how to answer the question, please share it!

Studying and learning outside of class

We only spend 3 hours in class each week together, and according to the Federal Credit Hour standard that means you should expect to spend at least 2 hours outside of class per week for each credit hour of class. Therefore the time you spend outside of class on this material is the single biggest component of the course! To help support you with this work outside of class, I will occasionally post videos of extra problems and proofs and will post selected homework solutions.

The time outside of class you spend learning and studying is how you develop your own perspective on the course material, deepen your understanding, and strengthen your skills. It is also important that you check in with yourself regularly on how the class is going, and part of your Effort grade will involve occasional reflection assignments.

Homework

There will be one homework assignment per week, due on *Tuesdays by 11:59pm* through Gradescope. I will not accept homework in person, you must scan it and turn it in online. Homework turned in after the due date will not be accepted (see the special arrangement section of the syllabus for extension policies). In order to make sure your work is organized well, please make sure you write legibly and label the problems in the same order as listed in the assignment. I will drop your lowest homework grade.

Homework gives you an opportunity to get feedback on your demonstrated understanding of course material outside of an exam. That said, you would probably prefer to make mistakes on the homework and learn how to fix them before a quiz or exam than make a mistake for the first time on a quiz or exam! This means homework will be most beneficial to you if you study the material and look at examples *before you start the related homework*, and push yourself to do the homework in a closer to exam-style environment! Try not to flip back to example problems unless you have given a problem a serious try without any help and you will find new problems start to get easier.

Your homework consists both of reading the relevant sections of the book and of doing the assigned problems. (Only the written work counts directly in your grade, but I expect you to do both.) *Start working on each problem set the same day it is assigned*; do *not* put it off until a night or two before it's due.

Homework grading will be more general feedback than quizzes and exams; this is intentional to help you feel more comfortable trying out the material for the first time. On quizzes and exams, you have had more time and practice with the material. After each homework assignment, I will post solutions to selected problems. *One the solutions have been posted, I will not accept that homework assignment late, even if you haven't used up your two allowed extensions.*

Working in groups and talking through your ideas is a great thing to do, and a skill that will be invaluable throughout your mathematical journey. However, the homework you turn in must be written *by you*. Copying or paraphrasing the work of others is plagiarism and is *a violation of the honor code*. This includes using ChatGPT, Chegg, CourseHero, Wolfram Alpha, and similar websites and programs. If you happened to work on an assignment with other people and you all came up with the same solution, please write the name(s) of the other students involved.

It has become very common for students to use GenAI (artificial intelligence or machine learning tools such as ChatGPT or Dall-E 2) on homework assignments in ways other than simply asking for answers to homework problems. This practice may

not be particularly helpful for preparing you for the quizzes or the exams, however, if you still decide to use it for homework, you must properly document and credit how you used it. This includes using it to brainstorm or generate code, outlines, or other text for you.

Sample Citation: Chat-GPT-3. (YYYY, Month DD of query). "Text of your query." Generated using OpenAI. https://chat.openai.com/
Note: Material generated using other tools should follow a similar citation convention.

Finally, if you talk to other students about the homework problems *do not give unsolicited answers to your classmates*. The ideal discussion of the homework is one where everyone involved has tried the problem already, and everyone in the discussion is given space to try out their ideas.

The software system LaTeX is the international standard for typesetting mathematics and scientific documents in many other disciplines. Once you get the hang of it, it will become much easier for you to type professional-looking mathematical documents, presentations, and posters. In order to give you an opportunity to practice, each homework assignment that you fully typeset in LaTeX will earn 2% extra credit on the whole assignment. Note that this cannot bring your homework grade over 100%!

I don't expect you to have ever used LaTeX before (or to have any experience with coding or typetting), so to get started here is a short 30 minute LaTeX tutorial. I strongly recommend using Overleaf to typeset your work; there are editors you can install on your computer, but they can sometimes be tricky to set up. There is also a homework LaTeX template on Moodle you can start with; the best way to learn LaTeX is to get in and start trying it!

Biweekly quizzes

Starting the second week of class, there will be a quiz every other week on Friday during weeks we don't have an exam. These quizzes are to provide a regular check in for both of us on how the material in the class is going for you. Since a semester has many twists and turns for everyone, I will drop your lowest *two* quiz grades.

You will be allowed one 8.5' x 11" page of notes (you can use the front and back of the paper) which you must turn in with your quiz. I strongly suggest making your own page of notes rather than sharing with another student, as making your own note sheet will help you organize your thoughts in the best way for you and usually helps you study.

If your total quiz average is higher than your lowest midterm grade at the end of the semester, I will replace that midterm grade with your quiz grade. However, you cannot replace your quiz average with an exam average.

Exams

There will be 2 midterm exams in class and one comprehensive final exam during final exam week.

Midterm Exam 1	Friday, October 11
Midterm Exam 2	Friday, November 15
Final Exam	TBA

An important part of learning the material in this course is gaining fluency in the concepts, computations, and proofs we are learning together. Being able to succeed on an exam in a timely fashion is part of being able to demonstrate that fluency. For example, if you wanted to become fluent in a new language, one of your goals would probably be to be able to have a conversation without having to refer to a dictionary for most of the words and grammatical structures during the conversation (this is one of the skills distinguishing different levels of fluency in a language by the ACTFL and CTFR scales). For this reason, the exams for this class will be closed book and closed note. Calculators, cell phones, laptops, ipads, etc. are not permitted in exams.

Additionally, the kind of deep studying required for an exam allows you to engage in the material in a different way and learn the material more thoroughly. Think about it this way: would you study harder for a class with no exams or a class with exams?

Finally, exams provide valuable feedback for both of us on specific things you are understanding and not understanding in this course, and can help both of us make an informed decision about how the class is going.

Office hours and Math Fellow Hours

Office hours with myself, the Math Fellows, and the Q Center are your time to have smaller group interaction and feedback with us to clarify concepts, ask questions, and get help if you are stuck on a problem. They will be most effective if you prepare for them by attempting the homework first, reviewing both your class notes and mine, and reading the textbook.

You can think about going to office hours like meeting with a coach or a trainer: we can demonstrate lifting a weight for you, but ultimately, if you don't also actually lift the weight yourself you won't get the benefits from lifting weights! Office hours

are for helping you navigate problem solving and learning the material in the course, which is not the same thing as simply telling you the solutions to homework problems. Office hours and math fellow hours supplement the rest of the components of this course, but they *cannot replace any other component of the course*.

Some of the things you can expect from the Math Fellows and QCenter Fellows

- Treating you with dignity and respect,
- Holding regular office hours,
- Supporting your problem solving and learning in this course with a student-centered approach,
- Asking you questions to pinpoint exactly where you need more support in this class, and providing that support based on their significant experience and expertise (this might not include giving you the final answer!)
- Their support will likely be a mix of helping with specific problems and more broad, structural advice about studying and engaging with the course; both will help support your success!
- They will be patient with you and respect the learning process.

Some of the things the Fellows will expect from you

- Treating them with dignity and respect,
- Trusting their experience and expertise and that you will try out the things they suggest you try,
- Assume they are acting with good intentions and in good faith and have been chosen as Fellows because they know what they are doing!
- You will be patient with them and respect the learning process.

My Office Hours

Monday 9-9:30am by appointment in person or zoom using Calendly
Monday 3-4pm drop-in hours
Wednesday 9-9:30am by appointment in person or zoom using Calendly
Wednesday 1:30-2:30pm drop-in hours
Thursday 2:30-3:30pm drop-in hours

Tuesdays are my research days and I am not available for meetings.

Hours with Math Associate Allison Tanguay at the QCenter

Starting week of September 8

Drop-In Hours in SMUD 208/208a

MWF 10am-12pm T/Th 2-4:30pm

Click here to make an appointment with her.

Math Fellows: Allison Klingler and Seth Yoo

Fellow Office Hours TBA

Starting week of September 8 in SMUD 208a

Mon 7:30-9pm Allison Klingler

Tue 7:30-9pm Seth Yoo

Thu 7:30-9pm Seth Yoo

Fri 7:30-9pm Allison Klingler

Required Text

All required textbooks at Amherst are now provided by the college! You will receive an email from customerservice@efollett.com notifying you that course materials are ready for pickup.

Please bring your Student ID to the textbook distribution center at the Alumni House to collect your course materials. If you drop the course on or before the last day of the add/drop period, you must return the print materials to the same Alumni House location.

Distribution Dates: August 27 - September 13
Location: Alumni House
Hours Monday - Friday, from 10 a.m. to 4 p.m.

Required text

Concrete Abstract Algebra: From Numbers to Gröbner Bases by Niels Lauritzen.

Add/Drop and Attendance Policy

It is very difficult to catch up in a math class if you miss classes, even the first couple of days of the semester. We will start out the semester with some tricky, abstract ideas and trying to catch up if you miss the topics the first time in class is unnecessarily difficult. If you're interested in adding this class, you should attend every class the first week.

Our time together in class is very important and there is quite a lot of data that attending class regularly strongly correlates with success in the course (see Credé, Roch, and Kieszczynka 2010).

As a result, I will take attendance in this class. If you have at least four unexcused absences (which is over 10% of our class meetings for the semester) you must schedule an individual meeting with me to discuss your absences. This may or may not be paired with a grade penalty.

If you come to class more than 15 minutes late without an excused reason, that will count as an unexcused absence.

The Amherst College Honor Code and Course GenAI Policy

The Amherst College Honor Code applies to this course. It is your responsibility and mine to be familiar with and uphold *all aspects* of this code, including the Statement of Intellectual Responsibility, the Statement of Respect for Persons, the Statement of Freedom of Expression and Dissent, and the Statement of Student Rights.

Generative AI will not be allowed on any in-class assignment and if used will be considered a violation of the honor code.

How to earn a specific grade in this class

Your final grade in the class will be computed by:

Effort	3%
Biweekly Quizzes	15%
Highest Midterm Exam	25%
Lowest Midterm Exam	20%
Final Exam	30%
Homework	7%

"Effort" is a combination of class attendance, class participation, and handing in problem sets. Final grades will be determined based on a holistic evaluation of your performance throughout the course. This may include, but is not limited to, mastery of both the proof-writing and computational components of the course, participation, improvement over time, and engagement with course materials. At the end of the semester, I as the instructor reserve the right to make adjustments to the final grade to reflect overall effort and learning in the course. Final course grades will be curved.

Any student who skips an exam for an unexcused reason or fails to hand in at least 50 homework problems *on time* over the course of the semester *automatically* gets an F in the class.

Special arrangements

Illness Policy

If you are showing symptoms of a contagious illness and/or testing positive for COVID-19 on a regular class day, please follow the Amherst College *protocols* and do not come to class. If it is an exam day, please email me immediately to schedule a makeup exam.

Extensions and Make-Ups

If for some reason you cannot hand in your homework in time or take a quiz on time, you can request up to TWO extensions during the term with no questions asked. You must contact me no later than the day before the due date to let me know. If you have a religious holiday on the same date as an exam in this course, let me know within the *first two weeks of the semester*.

Accommodations

I strive to support all students so please come meet with me if you have any questions or concerns about your engagement and success in this course. Students seeking general disability services and/or accommodations should contact Accessibility Services. You can reach them via email at accessibility@amherst.edu, or via phone at 413-542-2337. Once you have your accommodations in place, I will be glad to meet with you privately during my office hours or at another agreed upon time to discuss the best implementation of your accommodations. For more information, please visit the Accessibility Services website.

Electronics Policy

Please silence your phone and put it away during class. As many students find laptops distracting, if you plan to use a laptop during class please sit in the back row.

What we'll be learning about

The following schedule is tentative and may shift slightly as the semester develops. Nothing will be made due earlier than indicated but some things may be pushed back or eliminated altogether, depending on time. All changes will be announced in class and posted on Moodle.

Week	Topics we'll explore
Week 1	• Appendix A: Mathematical Foundations • Relations and orders • Division and congruence
Week 2	• Division and congruence • The Euclidean Algorithm • Sun-Tsu and Chin Chiu Shao's Remainder Theorem • Binary Operations • Definition of a group
Week 3	• Abelian groups • The group $\mathbb{Z}/n\mathbb{Z}$ • Composition tables/Cayley tables
Week 4	• Subgroups • Cosets • Normal subgroups

Week 5

- Group homomorphisms
- Kernel and image
- Quotient groups
- The isomorphism theorem

Week 6

- Generators of a group
- Cyclic groups
- Applications of group theory to number theory
- Midterm Exam 1

Week 7

- Finitely generated abelian groups
- Group actions
- Symmetric groups
- Theorems about permutations

Week 8

- Transpositions
- Orbits
- Stabilizers
- Burnside's lemma
- Sylow theorems
- Definition of a ring

Week 9

- Units and zero divisors
- Definition of an integral domain
- Definition of a field
- Subfields and extension fields
- Ideals
- Quotient rings

Week 10

- Principal, prime, and maximal ideals
- Ring homomorphisms
- Characteristic of a field
- Field of fractions

Week 11

- Unique factorization domains
- Prime and irreducible elements in a ring
- Midterm Exam 2

Week 12

- Euclidean domains
- Gaussian integers
- Polynomial rings
- Roots of polynomials over a ring

THANKSGIVING BREAK

Week 13

- Roots of unity
- Finite fields

Week 14

- Wrap up

FINAL EXAM WEEK

Important Dates

September 3	First Day of Classes
September 12	Last Day of Add/Drop
October 11	Midterm 1
October 14-15	Mid-Semester Break
November 15	Midterm 2
November 25-29	Thanksgiving Break
December 11	Last Day of Classes
December 12-15	Reading Period
December 16-20	Final Exams

This syllabus provides a general plan for the course; deviations may be necessary. You are responsible for all of the information in this syllabus, so please read it carefully and refer back to it regularly.